Vague Geographical Knowledge Management - A flow-chart based application to spatial information analysis

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Abstract

The issue of vagueness in using spatial information is becoming more important with the advancement of GIS technology and Spatial Decision Support

Systems. Vagueness is inherent in spatial knowledge due to our inability to make sharp and precise distinctions in our world. The concept of vagueness in spatial information systems deals with the evaluation of linguistic representations of spatial data.

This paper focuses on the consequences of incorporating vagueness into spatial models. Geographical notions of terrain features exemplify this approach. Vagueness in linguistic descriptors of terrain features is analyzed with MapModels, which is a flow-chart based easy-to-use programming language extending ESRI's ArcView® 3 GIS. Variations on modelling implicit and explicit vagueness in geographical data (e.g. mountainous slopes) are evaluated to comprehend the usefulness of vagueness as a source of information. Measures of fuzziness and MapModels are presented as means for advanced spatial knowledge management.

Keywords: Vagueness, Measure of Fuzziness, Fuzzy Logic, Linguistic Variable, MapModels, GIS, Mapping Vagueness

Introduction

Introducing fuzzy sets as an extension in modelling categories has drawn the attention on the discussion on different types of certainty as well as uncertainty. Fuzzy sets broadened the concepts on dealing with error and imprecision, which are issues well known to spatial analysis. Eliminating or tracking error and imprecision has been contributing to increase knowledge with respect to spatial phenomena. Notions in space and spatial terms became more precise in a complex decision making environment, too. The use of fuzzy sets in GIS applications triggered the question on what kind of uncertainty fuzzy logic is aiming at in the process of gaining precise knowledge. Following an approach discussed in (Spies 1993) ambiguity and vagueness are addressed by fuzzy methodology. Uncertainty in knowledge (i.e. ambiguity) occurs when values are associated with multiple attributes and no reliable decision criterion is available, e.g. "Is a particular elevation a 'hill' or a 'mountain'?". Uncertain knowledge (i.e. fuzziness or vagueness) is due to our inability to make sharp and precise distinctions in our world, e.g. "What *is* a 'mountain'?" as opposed to "What *is not* a 'mountain'?". Whereas uncertainty in knowledge is being dealt with by probability and possibility measures, uncertain knowledge is addressed by measures of fuzziness (Klir and Folger 1988).

This paper is focusing on the second type of uncertainty, which comes with the linguistic representations of geographical terms. Fuzziness or vagueness is seen as different from ambiguity and the problems of making decisions under uncertainty. It rather focuses on the problem of making precise distinctions in our world. Measures of fuzziness are used to develop examples on terrain features of slope which will illustrate the difference of modelling "How steep is 30%" from "How steep is steep?" referring to ambiguity and vagueness, respectively. Vagueness should not be “tolerated” or “eliminated” but rather be treated as a main source of “knowledge” as represented in the linguistic descriptors of geographic phenomena like “steep” or “flat”. In the next sections a logical background on modelling vagueness is developed: A measure of fuzziness is introduced enabling the user to model vagueness in GIS. MapModels is introduced as a sophisticated interface to do spatial analysis. The final part provides three examples on how explicit and implicit vagueness in geographical categories may be evaluated. Results of applying different parameters on measures of fuzziness and further research issues are discussed.

Vagueness, Fuzzy Sets & GIS

Vagueness is nothing special to geography since “vagueness is a pervasive phenomenon of human thought and language” (Varzi 2001). Vagueness is, nevertheless, a ubiquous and inherent feature in spatial knowledge on geographical phenomena. This is mainly due to the usage of linguistic terms in describing geographic reality introducing semantic aspects in the categorization of percepted, experienced or measured data and information. Vagueness is associated with the problem of making sharp distinctions in the real world. “Vague knowledge is not wrong per se, but it cannot be mapped using traditional logic” Russell is quoted in (Biever 1997). Black developed a “consistency profile” because “with the provision of an adequate symbolism the need is removed for regarding vagueness as a defect of language” (Black 1937). With the introduction of fuzzy sets in 1965 (Zadeh 1965) as the core of a multi-valued logical model we are able to use some of the information that comes with vagueness by applying that logical extensions to spatial terms. Vagueness is becoming a major source of information rather than another description of blurriness, error or other kind of imprecision. Vagueness or fuzziness “is not a burden that makes it tedious to know about anything but is an efficient way for humans to solve problems in their daily lifes and use expert knowledge appropriately” (Spies 1993).

The concept of truth is at the core of modelling fuzziness. The analogous meaning of a set membership to a truth value of a proposition offers new ways of dealing with vagueness that is focusing on the logical truth and its complement (Fig. 1). Fuzzy logic is just one example of an multi-valued logical concept that allows for modelling semantic aspects in vague categories. In classical set theory and logic we are limiting ourselves to only two truth values. A membership in a classical fuzzy set is equal to the truth value of a statement saying that ”‘value x is a member of set A’ is true”. GIS analysis using classical logic would assign “no data” to the membership value 0.8, which is further developed as an example in the application section.

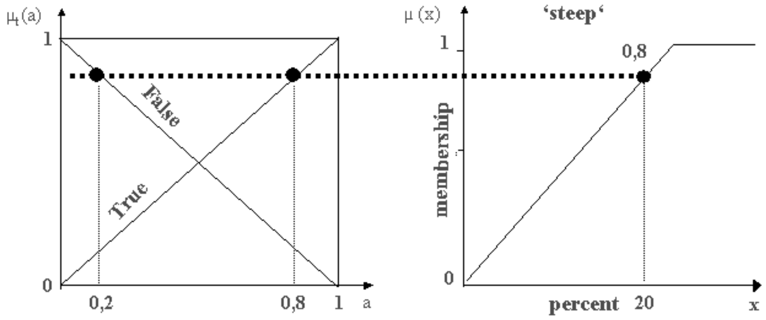


Fig. 1. The logical truth (left) of membership (right) in a fuzzy set "steep" (i.e. the truth value of a proposition '20% are steep' is 0.8)

Fuzzy logic introduces a number of truth values to a proposition. The truth value itself can be modelled by any kind of function and thus allows for dealing with a new level of "preciseness". The goal is not to find an "exact" truth value but one value that represents the mapped knowledge at best (Fig. 2). At first sight, this is intriguing of course and seems to make assigning a truth value an arbitrary task. Classical logic, however, is argued to be arbitrary in the first place due to its limitations in representing gradual transitions of truth. Fuzzy logic is adding to the transparency of logical modelling by being much more tolerant on different "truth" values that come with a complex spatial decision making situation. Classical logic in that part is not adjustable and thus not tolerant of any error, vague concept or imprecision of any kind. We argue this kind of "inexactness" being an enhancement to spatial knowledge modelling and also adding to the modelling of imprecision.

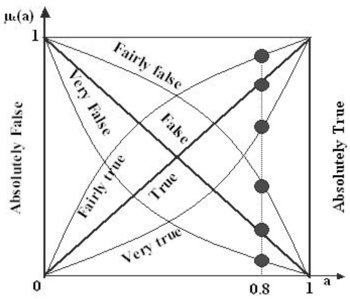


Fig. 2. Examples of logical truth functions (cit. Klir and Folger 1988)

Spatial categorization in GIS relies on classical logic. GIS use "objectified geographical entities", which are not necessarily the right means to model the "boundary poor empirical world", as Helen Couclelis put it (Couclelis 1993). Geographical categories on terrain features (e.g. slope) are, however, described by spatial properties like "steep". The question in this paper is is not directed towards finding more precise means to measure “steep” (“is it 28%, 29% or 30%?”) but to what extent “steep” is representing the information inherent in both the area of investigation and the knowledge of the GIS expert. In other words, we focus on how much a GIS expert is convinced that “steep” is the right descriptor of a spatial phenomenon thus expressing the quality of the linguistic descriptor. This “strongness” is argued to be modeled by the most simple formal operation on a fuzzy set, the complement. Furthermore, an example will be given on how much this complement is contributing to related concepts like the seemingly opposite of a spatial notion (“flat”). Different kinds of complements have been introduced, e.g. Kaufmann’s linear index of fuzziness (Tizhoosh 1997). In this paper we employ complements of the so-called Yager-class as referred to by George Klir and Tina Folger (Klir and Folger 1988).

The fuzziness we are dealing with is inherent in the linguistic category applied to geographical objects (slope) which we are using. The fuzziness is not based on the undecisiveness on whether 30% is steep or not. The question addressed is “How steep *is* steep?”. This implies that we regard the opposite of steep not being flat (or the problem of deciding whether a pixel belongs to a city or a suburb). The opposite of steep is not ‘flat’ but ‘not steep’, which in turn adds to the quality of describing the opposite. That is, all the possibilities of an area being ’flat‘ lie in the possibilities of logical modelling of ’not steep’.The differences set the frame for all possible logical solutions and thus provide information on the membership of a steep slope as well as a flat slope in a decision making process. Implicit and explicit vagueness that comes with fuzzy sets is used to model the strengthness or “knowledge” of a spatial problem or issue or category.

Measure (Index) of Fuzziness

The extension of set theoretical notions makes it necessary to extend GIS operations on spatial terms, too. The complement is the most self-evident (and simple) operation on a single fuzzy set. In classical set theory the complement would be 0 or 1. The content on information on vagueness is equal to the set membership. A fuzzy set, however, allows for a much more diverse set of operations by introducing multi-valued truth evaluations as stated in the the previous section. A measure of fuzziness in this logical context indicates the relationship between a set (‘steep’) and its complement (‘not steep’). Following (Klir and Folger 1988, p. 40ff) we employ a Yager-class of complements:

$$c_w(a) = (1 - a^w)^{1/w} \qquad (1)$$

where c_w assigns a membership value to $a \in A$ with $w \in (0,\infty)$ and A being a crisp or fuzzy subset (e.g. ‘steep’) of the universal set ‘slope’. Using parameter w allows to model different complements on a fuzzy set depending on its vagueness. Changing w results in a deformation of the function c_w. With $w = 1$ the function becomes the classical complement (i.e. $c_1(0.8) = 0.2$). The following discussion is based on this classical complement. For better readability the parameter *w* is omitted therefore throughout this section.

Given the relationship between a membership concept and truth values of a fuzzy proposition (see Fig. 1) it can be seen that this simple operation (see equation (1)) extends our ability to model the amount of contrariness which can be of value when used e.g. in GIS overlay operations. This general class of complements is used as a basis for aggregating the "lack of dictinction between a set and its complement" (Klir and Folger 1988, p. 140ff).

$$\delta_{c,A}(\mathrm{x}) = |\mu_{\mathrm{A}}(\mathrm{x}) - \mathrm{c}(\mu_{\mathrm{A}}(\mathrm{x}))| \quad \text{for all } \mathrm{x} \in \mathrm{X} \qquad (2)$$

where δ represents a distance measure between the set and its complement. $c(\mu_A(x))$ assigns a value to each membership grade $\mu_A(x)$ using equation (1). While δ expresses local (i.e. individual) differences, the total amount of contrariness between a set and its complement is defined as

$$\mathrm{D}_{\mathrm{c,r}}(\mathrm{A},\mathrm{A}^{\mathrm{c}}) = [\textstyle\sum_{\mathrm{x}\in\mathrm{X}} \delta_{c,A}^{r}(\mathrm{x})]^{1/\mathrm{r}} \quad \text{for finite sets} \qquad (3)$$

or

$$\mathrm{D}_{\mathrm{c,r}}(\mathrm{A},\mathrm{A}^{\mathrm{c}}) = [\int_{\mathrm{x}\in\mathrm{X}} \delta_{c,A}^{r}(\mathrm{x})\,\mathrm{dx}]^{1/\mathrm{r}} \quad \text{for infinite sets} \qquad (4)$$

where A^C denotes the complement of A produced by function *c* and $r \in [1,\infty)$. r relates to the metric concept used for modelling the indivdual differences. If, e.g. a Eucledian distance is used, r would be 2. Based on aggregating individual differences, a general measure of fuzziness *f* is defined as an index of vagueness (Klir and Folger 1988), (Benedikt and Nishiwaki 1998):

$$f : \mathrm{P}(X) \to \Re \qquad (5)$$

where P(*X*) denotes the fuzzy power set of *X* (i.e. the set of all fuzzy subsets of *X*). In this application *X* is the universal set of all slope-related linguistic terms. In our particular application *f* assigns a value *f*(A) to each fuzzy subset A of *X* in $\Re$, the set of real numbers, which characterizes the degree of fuzziness of A. The measure (index) of fuzziness using a Minkowski class of metric distances, e.g. Euclidean distances, has the following form:

$$f_{\mathrm{c,r}}(\mathrm{A}) = |\mathrm{X}|^{1/\mathrm{r}} - \mathrm{D}_{\mathrm{c,r}}(\mathrm{A},\mathrm{A}^{\mathrm{C}}) \qquad (6)$$

with |X| denoting the cardinality of the base set. The normalized version of $f_{c,r}$ (A) is

$$f^{\wedge}{}_{\mathrm{c,r}}(\mathrm{A}) = 1 - \frac{\mathrm{D}_{\mathrm{c,r}}(\mathrm{A},\mathrm{A}^{\mathrm{C}})}{|\mathrm{X}|^{1/\mathrm{r}}} \qquad (7)$$

so that

$$0 \le f^{\wedge}{}_{c,r}(\mathrm{A}) \le 1 \qquad (8)$$

For a detailed discussion on developing a more general class of fuzzy measures we refer to (Klir and Folger 1988). Applications using other classes of complements (e.g. Sugeno class of complements) or measures of fuzziness (e.g. quadratic index of fuzziness) (Tizhoosh 1997) are not discussed in this paper.

A measure of fuzziness is adressing precisely the concept of uncertainty or fuzziness, which makes the use of crisp sets distinct from fuzzy sets. For crisp sets $f\hat{}_{c,r}$ is 0, in the fuzzy case the value increases as the mean distance between a set and its complement decreases. $f\hat{}_{c,r}$ equals 1 with all membership values being 0.5 throughout X. In the following section this measure of fuzziness is applied to geografic notions within a GIS analysis environment.

Application - Study Area

The study area is located in the Austrian Alps (Fig. 3). The region in Tyrol and Vorarlberg along the border to Switzerland is known as Silvretta Mountains. The area is digitally stored as a grid (raster) image. The used DEM (digital elevation model) has the size of 100km² in reality. These 100km² are divided into 1024x1024 pixels. One pixel covers about 100m² ground area and the value of the elevation is saved in each cell (pixel). The study area is situated in a height of 1746 to 3306m above Adrian sea level. The used raster image was interpolated from digitised map data of the BEV (Federal Office for Metrology and Surveying). So the basic data of the study is the most accurate available in Austria (Hemetsberger et al. 2002)

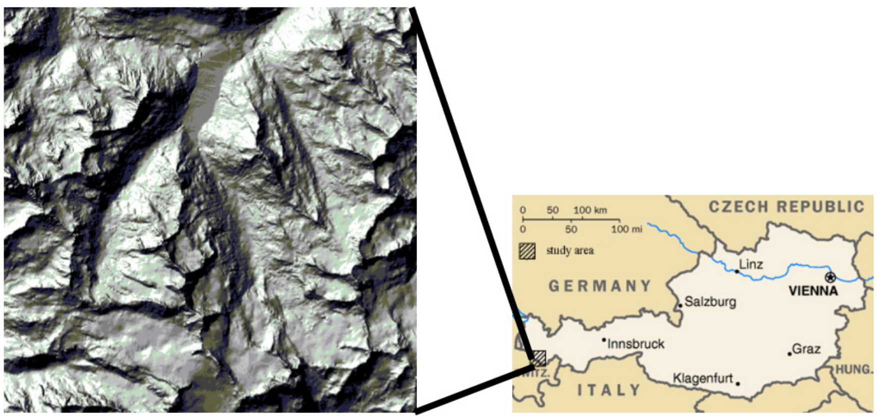


Fig. 3. The study area

Application - Software

MapModels is a flexible tool for explorative spatial data analysis. It has been developed at the Regional Science Institute of the Vienna University of Technology with the intention to bridge the gap between spatial decision analysts and computer programmers (Riedl et al. 2000). MapModels is a visual programming language based on the widespread desktop GIS ArcView 3. It supports the development and implementation of analysis procedures based on flowchart representations in a very intuitive and user-friendly manner. It is particularly suited for extended decision making with fuzzy set modelling of geographical notions.

Flowcharts are used for the visualization of models and analysis processes in a wide range of applications. Normally this kind of graphic representation is simply focused on the illustration of the model structure and information flow but doesn't directly control the underlying processes.

Within MapModels the nodes of a flowchart are in fact active elements of the model. They provide a visual encapsulation of real analysis procedures and data objects where input data and analysis operations are represented by labelled icons connected by edges which characterize the dataflow (Fig. 4).

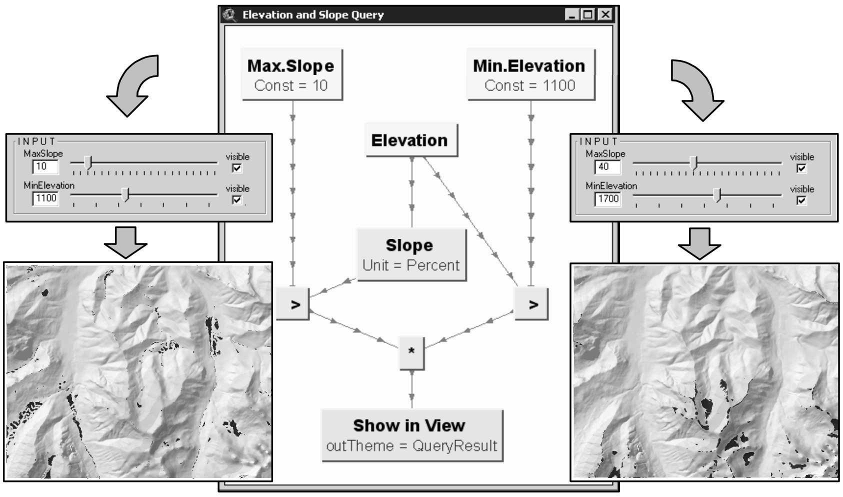


Fig. 4. a simple spatial query: "find all relatively flat areas with an elevation higher than a given threshold" (center: MapModel; left: slope<10% and elevation>1100m; right: slope<40% and elevation>1700m with corresponding results)

Since MapModels flowcharts contain executable code, the specification and the implementation of the analysis model is just one single step. It is a kind of a drawing-process where flowchart elements are inserted into the model environment and connected by means of drag-and-drop operations with the mouse.

Currently a basic function library is available which contains a steadily increasing number of flowchart elements for a wide range of analysis operations including e.g. the application of fuzzy logic (Benedikt et al. 2002). Standard MapModels-users usually will apply functions out of this set. Users with basic Avenue™ programming skills can extend and/or customize the existing set according to their specific needs with low effort.

The recently implemented option of "nesting" models enhances the flexibility of MapModels (Riedl and Kalasek 2002). This concept is very similar to the routine-subroutine design of conventional programming languages. A submodel is defined as a collection of MapModels-functions which act as a single flowchart element when used within other models. An input/output interface has to be defined for each submodel in order to address the submodel on higher levels of the model (Fig. 5).

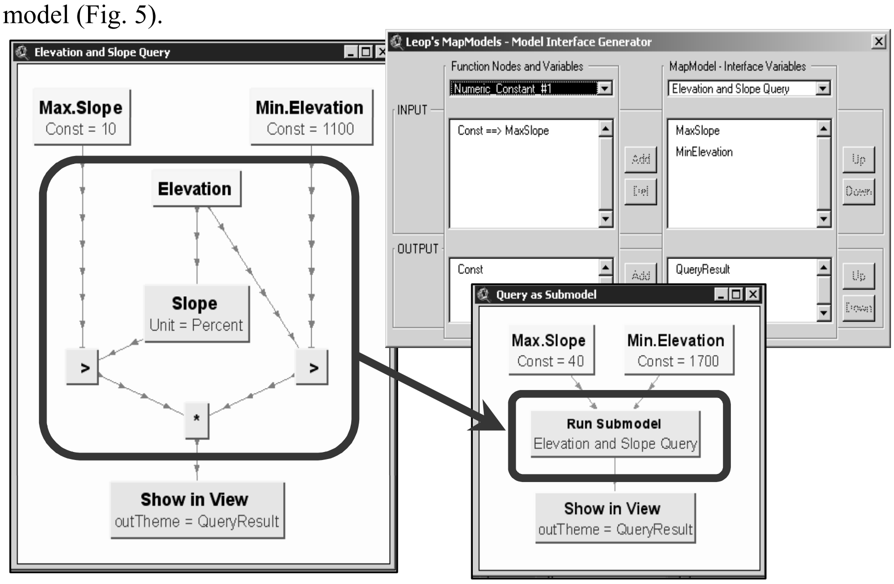


Fig. 5. Concept of submodels: the flowchart inside the dark box (left model) is merged by means of a specialized dialog box (model interface generator, upper right picture) in a single node (lower right model)

This modular design enables users to build customized basic analytical units for their applications and reuse them wherever they want and need. This is especially suitable for tasks that are performed repeatedly using different parameters. Compared to "All-In-One" concepts (i.e. the whole analysis model is represented within a large single model) the modular approach in the recent version 2 of

MapModels[1] helps to keep models small and self-explanatory and avoids redundancies within the whole analysis-structure as well (i.e. duplication of programming code).

Application - Results

The first MapModel (Fig. 6) is designed to show the impact of implicit vagueness on visualizing uncertain knowledge in GIS layers that eventually become part of spatial decision making systems. The fuzziness is reduced automatically as a linguistic hedge (very) is applied. This is due to maximizing the distance between elements in 'steep' and 'not steep' just by using a different membership function on steep. Note, that there is no vagueness in a crisp set 'steep' (all values are 0). Due to the complete GIS functionality of MapModels the results of applying MapModels are shown as GIS layers in Fig. 7. We can see the vagueness of steep being very low at the ground and on top of the mountains, whereas the vagueness in between is pretty high related to the uncertainty that comes with modelling slopes. Note: The results do *not* refer to the transition zones between steep and more or less steep (or city and suburb and similar investigations of transition zones) but focus uniquely on the logical implication of memberships as represented by the linguistic variable steep (or city, respectively). The top is considered definitely steep. The grounds are definitely not steep. But in between the implicit amount of vagueness gives the opportunity for people (or in logical models) to discuss whether its steep or not by making explicit use of vaguenss and measures of fuzziness, which is seen as a major advancement of using fuzzy sets in representing geographical notions. (Benedikt et al. 2002). The calculation steps based on equations (1) to (7) are used to measure fuzziness or vagueness. The results are summarized in Table 1.

[1] A trial-version of the of MapModels (as an Extension for ArcView 3 GIS) including examples, literature and documentation can be downloaded on the authors webpage "*http://www.srf.tuwien.ac.at/MapModels/MapModels.htm*".

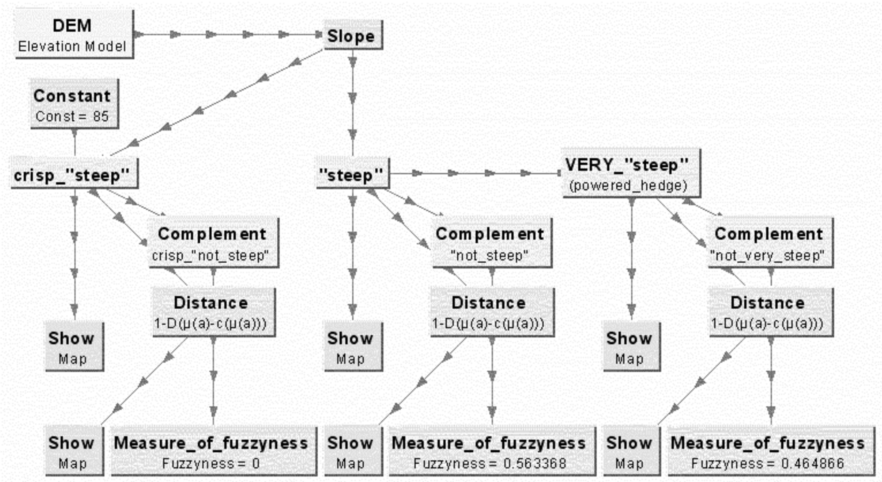


Fig. 6. MapModel of implicit vagueness of *steep* (crisp), *steep* (fuzzy) and *very steep* (fuzzy)

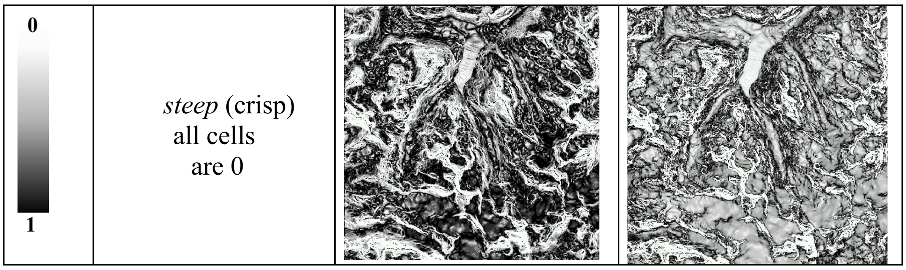


Fig. 7. Mapping implicit vagueness of *steep* (crisp), *steep* (fuzzy) and *very steep* (fuzzy)

Table 1. Degree of fuzziness in linguistic categories by using classic complements

Measures of fuzziness: $c_w(a) = (1-a^w)^{1/w}$, w = 1			
	Crisp steep	steep	Very steep
r= 1	0.00	0.56	0.46

Based on the first example we extend our modelling efforts towards making explicit use of the complement (Fig. 8). In the first example we just took the complement “as is”. Now we draw our attention to explicitly changing the truth functions (see also Fig. 2). In Fig. 8 this is indicated by the model nodes “How vague is…”. Using explicit fuzziness extends the possibility to explore the vagueness using the concept of a complement as a measure of fuzziness with respect to a particular situation. Within MapModels we can interactively change our opinion on the (un)certainty of 'steep (crisp)', 'steep (fuzzy)' or 'very steep (fuzzy)'. In addition our change of attitude can be seen immediately on maps. Note: In case of a crisp categorization of steep the results of using explicit vagueness by modelling the complement are "No data" (Fig. 9). Therefore crisp sets are suboptimal models to handle uncertainty in geographical analysis when vagueness comes into the picture.

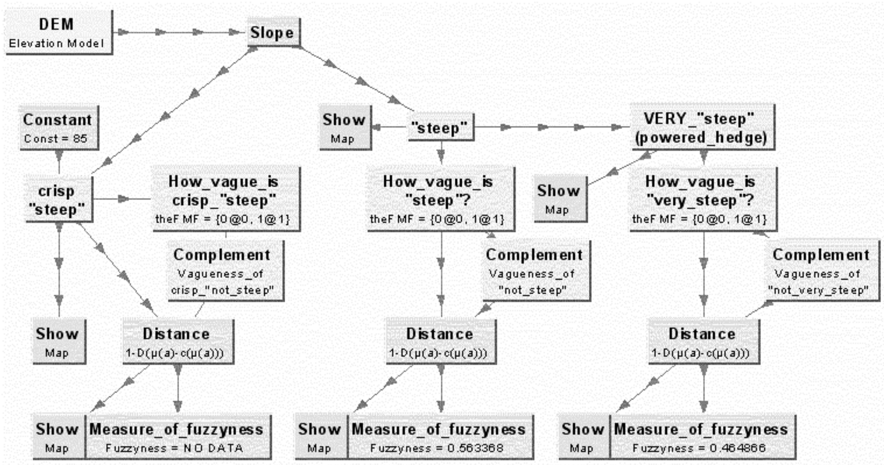


Fig. 8. MapModel of explicit vagueness of *steep* (crisp), *steep* (fuzzy) and *very steep* (fuzzy)

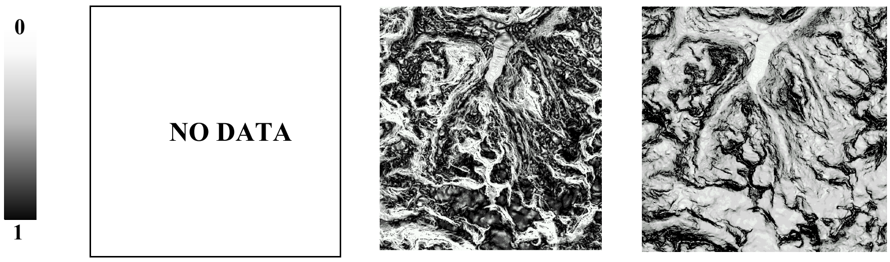


Fig. 9. Mapping explicit vagueness of steep (crisp), steep (fuzzy) and very steep (fuzzy)

Based on these applications the analysis of vagueness in a linguistic variable can be generalized to address the overlay of different fuzzy sets representing different variables. The idea behind this generalization is using explicit vagueness on a fuzzy set like *steep* to broaden the solution space of *flat* by exploring the relations between *non-steep* and *flat* (Fig. 10). The mapped results (Fig. 11) show the alternative approach of how another linguistic variable (fuzzy set) *flat* is contributing to the reduction of vagueness of another set (*steep*) by using the complement and employing different r as in (3). Table 2 is summarizing the measures of fuzziness achieved with different r.

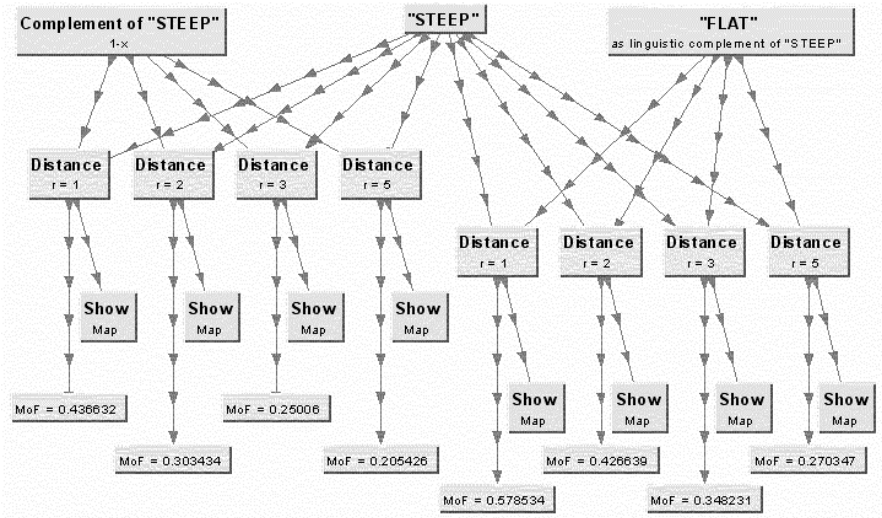
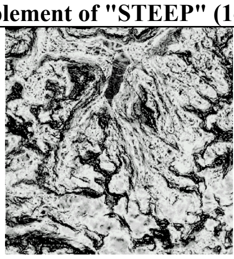
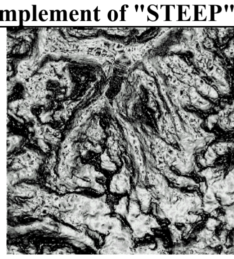
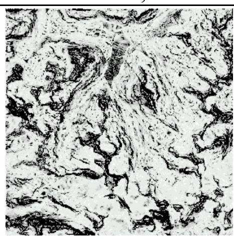
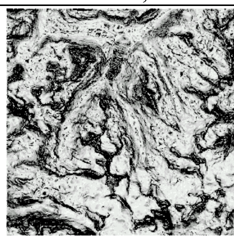


Fig. 10. MapModel of calculating (explicit) measures of vagueness for different measures of distance on arithmetic versus linguistic complements

	"NON-STEEP" as arithmetic complement of "STEEP" (1-x)	**"FLAT" as linguistic complement of "STEEP"**
r=1	 MoF = 0,437	 MoF = 0,579
r=2	 MoF = 0,303	 MoF = 0,427

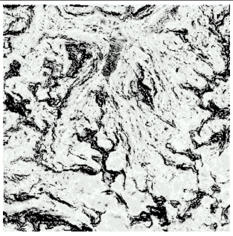
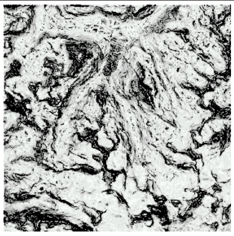
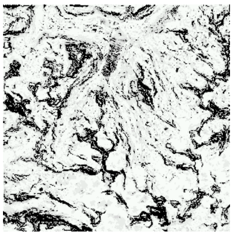
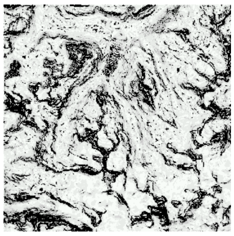
r=3	 MoF = 0,250	 MoF = 0,348
r=5	 MoF = 0,205	 MoF = 0,270

Fig. 11. Mapping vagueness of steep (fuzzy) by using explicit vagueness and complements; effect of complement and distance measure on the measure (index) of fuzziness expressed by equation (7)

In general, using a higher *r* results in a set with more precise boundaries, which is not surprising, since the difference between a set and its complement is getting larger, that is, the set elements share a greater distance. What is intriguing, though, of this kind of application is the possibility of using a logical model that allows for greater variations in modelling linguistic descriptors of geographical data rather than focusing on a single arithmetic calculation. Fuzziness of a variable contributes to a better understanding of another linguistic variable or at least allows for making differences in fuzzy set overlays more visible.

Table 2. Degree of fuzziness in linguistic categories by using steep/non-steep and flat

Measures of fuzziness: $c_w(a)= (1-a^w)^{1/w}$, w = 1		
	Steep (non-steep)	Flat (non-steep)
r = 1	0.437	0.579
r = 2	0.303	0.427
r = 3	0.250	0.348
r = 5	0.205	0.270

Conclusion

The overall goal of this paper was to explore the power of multi-valued logic in modelling semantic aspects of linguistic expressions used to describe spatial categories. Our applications focus on information which is argued to be inherent in linguistic terms and variables respresenting uncertain spatial knowledge. The truth values of slope measurements being "steep" and "non-steep" at the same time are examined by employing measures of fuzziness. To put the focus on logical truth of building linguistic categories offers possibilities in modelling semantic interpretations of data. The results do not refer to the transition zones between steep and more or less steep, or city and suburb or forest and meadow but focus uniquely on the logical modelling of memberships as represented by linguistic terms and variables such as steep, city, forest etc. This paper also shows the immediate impact on resulting maps by using MapModels, an easy-to-use modelling language.

The main issue of using fuzzy sets and MapModels is to make uncertain knowledge "visible" or "explicit", not to eliminate it. So far, uncertain knowledge has been eliminated (i.e mostly ignored) or treated as statistical error in general. Using MapModels we incorporated fuzziness to the spatial modelling process. Our models do not provide unique solutions which are valid in every instance but help to add more transparency in a complex decision making environment with many actors expressing different opinions.

This work and the application's results are meant to stimulate research on a fuzzy spatial logic regarding issues like tracking truth values along the overlay process in GIS analysis. Different logical models of the complement may also lead towards a greater flexibility. The results are promising in making advanced use of measures of fuzziness and in contributing to a better understanding of complex spatial phenomena.

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